The Taxation of Life Annuities Under Adverse Selection

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Abstract

This paper studies how annuities should be taxed in a model à la Mirrlees (1971) in presence of adverse selection and a positive link between income and longevity. An annuity tax can address the adverse selection problem by subsidizing small annuities (purchased by low income groups) and taxing large annuities (purchased by high income groups). Numerical simulations suggest that the degree of progressivity of taxation is significant and increases when annuitants get older.

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1 Introduction

Concerns over the future of public pension systems have led many governments to promote the development of private life annuity products by means of tax incentives. Whitehouse (1999) shows that most developed countries exempt from income tax either the contributions during the accumulation period or the benefits during the payout phase. Antolin et al. (2004) find that the first option is the most common regime in 17 OECD countries. Both options alter the post-tax rate of return to annuities, albeit in a different way. As individuals generally pay a lower marginal income tax rate while retired than in work, a tax-deferral policy tends to pull up the rate of return. Yet observed tax treatments are difficult to assess on the ground of economic principles.

Those fiscal exemptions raise a number of important policy issues which are more broadly related to the debate on the taxation of saving. Should the government tax or subsidize the returns to annuity ? Should the taxation be progressive in order to redistribute income ? Considering that developed economies already achieve redistributive goals through a personal income tax, is there any complementary role for a tax on annuity ? As regard to saving, the literature generally concludes that the taxation should be avoided. This statement can be traced back to the influential paper of Atkinson and Stiglitz (1976). They show that indirect taxation is needless when a government can use a non-linear income tax and utility functions are weakly separable between goods and leisure. In particular, the objective of redistribution is better achieved by an income tax alone. Since a tax on saving is equivalent to a commodity taxation which varies over the life cycle of the agent, this result extends to the taxation of saving as well.

Few studies exist however which look at whether this result applies to life annuities. Private annuity markets are indeed a distinctive segment of capital market. The return involves the expected mortality rate of the annuitants. Since it is generally not observed by insurance companies, this leads to an adverse selection problem. Moreover, average longevity tends to increase with income (see e.g. Deaton and Paxson, 2001). Both features justify why the analysis of annuity taxation deserves a separate analysis.

This paper studies how annuities should be taxed in a model à la Mirrlees (1971) with a continuum of skills, one working period and many retirement periods. It presents two arguments in favor of a taxation of life annuities. First, the taxation should address the adverse selection problem that plagues the annuity market. Indeed, the impossibility to extract or exploit information about individual mortality rates leads insurance companies to offer a common rate of return to all their customers. Compared with a first best economy, it follows that the market price of annuities is too high for the short-lived agents and too low for the long-lived individuals. In this context, the government can restore actuarial fairness by setting a corrective tax schedule on annuities.

A second argument for annuity taxation comes from redistribution purposes. It relies on the fact that, as the rich are more likely to attain old age, they benefit from a longer stream of annuities in average. A government can then reduce lifecycle inequalities by taxing annuities insofar as they signal consumption by high incomes.

The first argument considered in isolation implies a progressive taxation of annuities. The second one (annuities as luxury goods) is shown to lead to a positive yet regressive tax schedule. The model cannot therefore determine whether the overall effect leads to a progressive or a regressive tax. Next, I turn to a calibrated version of the model. Numerical results suggest that the level of taxation on annuities is progressive, significant and increases when annuitants get older. Low annuities are subsidized and large annuities are taxed.

This is not the first model which addresses the issue of annuity taxation. Sheshinski (2006) applies the theory of optimum commodity taxation to the pricing of annuities and shows that, under utilitarianism and symmetric information, a negative correlation between survival probabilities and incomes leads to subsidization of individuals with high survival probabilities. The results are however less clear-cut with adverse selection and a positive correlation between survival probabilities and incomes, which are two central assumptions of this paper. Saez (2002) introduces labor supply and unobservable productivities in a two period model. He assumes that the discount rate is positively correlated with skills. As a higher discount rate produces the same effect on future marginal utility than a longer life expectancy, his rationale for taxing saving is close to the luxury good argument presented in this paper for the taxation of annuities. Contrary to Saez, the paper by Brunner and Pech (2008) explicitly focuses on the annuity market in a model with two types of productivity and a single retirement period. They find that annuities consumed by the most productive agents should be taxed. This result is corroborated by the present model in a more general framework. The existence of a continuum of workers and several periods of retirement allow to address additional issues like the optimal shape of the tax schedule and how it evolves with the age of retirees. The model also provides quantitative insights about the degree of tax progressivity.

The paper proceeds as follows. Section 2 lays out the basic setup of the economy. Section 3 presents some properties of the income and annuity taxation system. In Section 4, the parameters of the model are calibrated and quantitative results are presented. Section 5 concludes.

2 The model

Let us consider an economy with n periods and a continuum of consumers whose productivities (or skills) w are spread over the continuum $W = [\underline{w}, \overline{w}[$ according to the distribution function $F(.)$ and the density function $f(.)$. The first period is a working period during which agents choose their labor supply L. The remaining dates are retirement periods. Consumption $C = (c_1, c_2, ..., c_n)$ takes place at each date if the consumer survives until then. Let $\pi_i(w) > 0$ denote the survival probability at age i of an individual w conditional on being alive at date 1. All agents are alive during the working period ($\pi_1(w) = 1$). It is assumed to be an increasing function of productivity: $\pi_i'(w) \geq 0$, $i = 2, ..., n$.

Individuals are characterized by a utility function $U(C, L, w)$ which takes a standard time separable form:

$$U(C, L, w) = \sum_{i=1}^{n} \beta^{i-1} \pi_i(w) u(c_i) + v(L) \tag{1}$$

where β is the discount factor, u and v are respectively period utility and disutility of work with the usual concavity and continuity properties. It is additively separable between consumption and leisure ($U_{Li} = 0$, $i = 1, ..., n$ where U_{Li} is the cross derivative between labor and consumption at date i) but not between consumption and skill since the latter affects survival probabilities ($U_{wi} > 0$, $i = 2, ..., n$ and $U_{w1} = 0$).

The uncertainty of survival calls for the purchase of annuities which deliver an income when subscribers are still alive in exchange for their wealth upon death. In the absence of a bequest motive or uninsurable risk, agents fully annuitize their wealth (see Yaari, 1965). Hence agents simply consume their net-of-tax annuity while retired. For simplicity, there is no minimal annuity provided by a social security program.

Labor and annuities are taxed by way of separate non-linear schedules in an economy à la Mirrlees (1971). The key assumption is that the government cannot observe separately the labor supply and the wage rate. It is thus restricted to setting taxes as a function only of earnings or annuities. Let $T(wL)$ and $t_i(c_i)$ be respectively the earning tax and the annuity tax at age i. Only the structure of commodity taxes, and not their level, constitutes an independent policy instrument. A uniform commodity taxation can be replicated by an appropriate adjustment in the income tax schedule. The commodity tax rate on first period good is therefore set equal to zero: $t_1(c_1) = 0 \; \forall c_1$.

The functioning of the annuity market is affected by an adverse selection problem. Insurance companies cannot observe the expected longevity of each insured and are bound to offer the same rate of return to all annuitants. As a result, the price of future consumption is too high for agents with low life expectancy and too low for individuals with better longevity prospects. The annuity market equilibrium is defined as in Abel (1986) by assuming that insurers cannot offer different annuity rates of return based on quantities purchased. This assumption is realistic as savers can divide their saving between different insurers in case of price discrimination. In addition, it is assumed that insurance companies offer separate contracts for each period of retirement. Let us define the (pooling) rate of return Q_i between date 1 and date i for a consumer still alive after i periods. In order to fund consumption c_i at age i, the insurance company collects during the working period from individual w the sum $(c_i + t_i(c_i))/Q_i$ and delivers the annuity $a_i = c_i + t_i(c_i)$ at age $i > 1$. From the company's perspective, it is liable to the expected sum $\pi_i(c_i + t_i(c_i))/R^{i-1}$ where R is one plus the safe rate of return. The zero profit condition in the insurance market with unobservable survival rates leads to the equality of the two expected cash flows for the whole market:

$$\int_W (c_i + t_i(c_i))/Q_i dF(w) = \int_W (c_i + t_i(c_i))\pi_i/R^{i-1} dF(w)$$

This defines the price of annuities in a pooling equilibrium. It is a weighted average of the population's survival probabilities, the weights being the (equilibrium) demands for annuities:

$$Q_i = \frac{\int_W (c_i + t_i(c_i))/dF(w)}{\int_W (c_i + t_i(c_i))\pi_i/R^{i-1} dF(w)} \tag{2}$$

Given market rates of return Q_i, the programme of the consumer w is given by:

$$\begin{gathered} \max U(C, L, w) \\ \sum_{i=1}^{n} (c_i + t_i(c_i))/Q_i = wL - T(wL) \end{gathered}$$

with $Q_1 = 1$. Consumers reach a level of utility $\mathcal{U}$ which ultimately depends on their wage rate:

$$\mathcal{U}(w) = U(C(w), L(w), w)$$

where $C(w)$ and $L(w)$ maximize utility given the resource constraint.

The government sets the income and annuity taxes $T(wL)$ and $t_i(c_i)$, $i = 2, ..., n$, by maximizing the integral over the population of a concave function of individual utilities $\int_W \Psi(\mathcal{U}(w))dF(w)$, subject to an aggregate budget constraint in which PS is an exogenous amount of public spending:

$$\int_W \left[\sum_{i=1}^{n} \pi_i t_i(c_i)/R^{i-1} + T(wL) \right] dF(w) = PS, \tag{3}$$

and subject to the constraint that individuals optimize in their choice of labor supply given the relationship between work and after-tax income (see appendix 1 for more details).

3 Analytical results

I begin by presenting the optimum income tax formula (see appendix 1 for the details):

$$\begin{aligned}
\frac{T'(wL)}{1-T'(wL)} &= A(w)B(w)D(w) \qquad (4)\\
A(w) &= 1+1/\varepsilon(w)\ ; \qquad \varepsilon(w)=U_L/LU_{LL}\\
B(w) &= \frac{1-F(w)}{wf(w)}\\
D(w) &= \frac{U_1}{1-F(w)}\int_w^{\overline{w}}\left(1/U_1-\frac{\Psi'}{E(\Psi')}E(1/U_1)\right)dF(z)
\end{aligned}$$

$\varepsilon(w)$ is the uncompensated elasticity of labor supply, U_1 the derivative with regard to consumption during the working period and $\Psi' = \Psi'(\mathcal{U}(w))$ is the marginal valuation of utility taken at the optimum. The intertemporal nature of the problem does not change how the factors usually found in the literature enter the income tax formula in a static framework. The labor elasticity ε negatively affects the marginal tax rate as it reflects how much labor supply will be reduced following an increase of the marginal rate. The third term represents the benefit from raising additional resources in terms of reduced inequalities and dispersion of marginal utilities. The second term $B(w)$ weighs the importance of those two effects (see also Diamond, 2003).

The marginal annuity tax at date $i > 1$ takes the following form at the optimum (see appendix 2):

$$\begin{aligned} 1 + t_i'(c_i(w)) &= H_i(w) \left(1 - \eta_i(w) B(w) D(w)\right)^{-1} \qquad (5) \\ H_i(w) &= \frac{\pi_i(w) Q_i}{R^{i-1}} \\ \eta_i(w) &= \frac{w \pi_i'}{\pi_i} \end{aligned}$$

where $B(w)$ and $D(w)$ are defined in Equations (4). η_i is the elasticity of the survival probability at age $i > 1$ with regard to skill w. In the special case with homogenous mortality rates ($\pi_i(w) = \pi_i$), the elasticity η_i is equal to zero and the rate of return Q_i simplifies to R^{i-1}/π_i. In that case, marginal tax rates on annuities are set equal to zero and annuity taxes can therefore be suppressed following an appropriate reformulation of the income tax, as initially shown by Atkinson and Stiglitz (1976).

Eq. (5) shows that the tax rate is the product of two terms $H_i(w)$ and $\left(1 - \eta_i(w) B(w) D(w)\right)^{-1}$. The first term deals with the adverse selection problem. To understand the intuition, consider the Euler equation of individual w:

$$\frac{u'(c_1(w))}{\beta^{i-1} u'(c_i(w))} = \frac{\pi_i(w) Q_i}{1 + t_i'(c_i(w))}$$

Absent an annuity tax ($t_i' = 0$), the actuarial rate of return of annuities $\pi_i(w) Q_i$ would be an increasing function of the survival probability, entailing a distortion of the consumption profile and redistribution from the short-lived to the long-lived. When one plus the marginal annuity tax is reduced to the first term $H_i(w)$, the tax-adjusted actuarial rate of return boils down to the intertemporal marginal rate of transformation which would prevail in a symmetric information economy:

$$\frac{u'(c_1(w))}{\beta^{i-1}u'(c_i(w))} = R^{i-1}$$

The term $H_i(w)$ therefore eliminates the adverse selection effect by applying an increasing marginal tax rate on annuities. $H_i(.)$ satisfies the following property (proof in appendix 3):

Lemma 1. Let us define the level of skill $\widetilde{w} \in]\underline{w}, \overline{w}[$ satisfying:

$$\pi_i(\widetilde{w}) = \frac{\int_W (c_i(z) + t_i(c_i(z)))\pi_i(z)dF(z)}{\int_W (c_i(z) + t_i(c_i(z)))dF(z)}$$

If $\eta_i(w)B(w)D(w) = 0$ in Eq. (5), $t_i'(c_i(w)) \geq 0$ for $w \geq \widetilde{w}$ and $t_i'(c_i(w)) < 0$ for $w < \widetilde{w}$.

Hence, taken in isolation, the first term $H_i(w)$ implies that the marginal tax rate on annuities is positive for agents whose skill is greater than the threshold $\widetilde{w}$ and is negative for agents whose skill is less than $\widetilde{w}$. Hence, the annuity of the poorest should be subsidized.

The second term of the product in Eq. (5) $(1 - \eta_i(w)B(w)D(w))^{-1}$ reflects the government's concern about income inequality. Since the scope of the earning taxation is limited by disincentive effects on labor supply, the government uses annuity taxation as a complementary tool. The more productive agents tend to live longer and therefore value more retirement consumption ($U_{iw} > 0$). The government can then reduce life cycle inequalities by taxing annuities, identified as a luxury good. Hence the greater the elasticity $\eta_i(w)$, the higher the tax rate on annuities *ceteris paribus*.

How does the tax rate on annuities vary with skill ? A first hint can be given by expressing the annuity tax schedule as a function of the income tax rate (see Appendix 2):

$$1 + t_i'(c_i(w)) = \frac{\pi_i(w) Q_i}{R^{i-1}} \left(1 - \frac{\eta_i(w)}{1 + 1/\varepsilon(w)} \frac{T'(wL)}{1 - T'(wL)}\right)^{-1} \tag{6}$$

The first term of the product (the actuarial adjustment effect already described) is increasing with skill whereas the second term (the luxury good effect) varies in the same direction than the marginal tax rate of income $T'(wL)$ (assuming that the elasticities η_i and ε do not vary with skill). Since the literature generally finds that $T'(wL)$ is decreasing over the main part of the income distribution (except in some cases at the two extremes, see Seade (1977) and Diamond (1998)), the marginal tax rate on annuities is decreasing as well, implying a regressive tax schedule. The overall effect on the slope of t_i' is therefore undetermined.

Eq. (6) is consistent with the results of Brunner and Pech (2008) who study a similar model with two levels of skills. They find that the sign of taxation is undetermined for the less productive agents and unambiguously positive for the most productive ones. Eq. (6) shows that their last result is only a local one as it only applies to agents endowed with the highest level of skill and for whom marginal taxation on income is zero when the skill distribution is upper bounded.

Note finally that the absolute value of the marginal tax rate on annuities can be potentially very high if the term $(\eta_i/(1+1/\varepsilon))(T'/(1-T'))$ gets close to unity. This may entail an interior kink in the budget constraint of the consumer and consequently a gap in the distribution of consumptions. Some individual second-order conditions would break in this case. Such a possibility is not explored further at this stage. Instead, it will be checked in the simulation exercise that optimal solutions lead to increasing earnings and consumption with skill, which is a necessary and sufficient condition for individual second-order conditions (Mirrlees, 1971).

4 Numerical simulations

The aim of this Section is to provide some quantitative assessments about the general shape of the annuity tax schedule.

4.1 Calibration

The maximum age of survival is set to 100. Optimal rates simulations are performed using intertemporal utility given in Eq. (1). Period utility is characterized by constant relative risk aversion: $u(c_i) = c_i^{1-\sigma}/(1-\sigma)$. The intertemporal elasticity of substitution $1/\sigma$ is taken to be 0.5, which is in line with common practice in calibrated macroeconomic models.

Labor disutility takes an exponential form:

$$v(L) = -\frac{\gamma^{-1/\varepsilon}}{1+1/\varepsilon} L^{1+1/\varepsilon}$$

Despite a sizeable literature on the labor supply response to changes in the net-of-tax wage, a fair amount of uncertainty persists about the precise empirical value taken by the labor supply elasticity ε. Results for the French population find values between 0.1 and 0.2 for men and an average of 0.5 for married women (see Bourguignon and Spadaro (2007) for references). An average value of 0.3 will be chosen for the model. The riskless interest rate is 4%. The subjective discount rate is chosen such that $\beta = 1/R$, implying that a constant consumption profile across time would prevail in a first best environment. The values chosen for those three parameters (σ, ε and β) are admittedly somewhat arbitrary. Appendix 5 shows however that the numerical results described below are robust to changes in the value of ε and β, and to a lesser extent to variations of σ.

The distribution of wages w are taken from Bourguignon and Spadaro (2007) whose estimates are based on a French dataset for the year 1995 and smoothed by kernel techniques (see Fig 1). The right tail of the distribution has been truncated so that the maximum wage is ten times the minimum wage observed in the distribution. The existence of an atom of non workers is also assumed at the bottom of the distribution. They represent 10% of the working force.

For simplicity, a utilitarian social welfare criterion is chosen for the government (that is $\Psi' = 1$). Redistribution takes place in the model through a guaranteed income level (equal to $-T(0)$) that is taxed away as earnings increase. It is set such that the ratio of government spending PS to aggregate production is equal to 0.3.

Inspection of the budget constraints shows that the income tax and the annuity taxes are not independent tools of redistribution (even though marginal taxes are). Only the intertemporal sum of taxes $T(wL) + \sum_{i=1}^{n} t_i(c_i)/Q_i$ matters in the consumers' intertemporal budget constraint. Hence, without loss of generality, the overall tax transfer to the less skilled agents is restricted to the guaranteed income level, meaning that annuity taxes are all set equal to zero for this group: $t_i(c_i(\underline{w})) = 0$, $i > 1$. Assuming a non-zero annuity tax would lead the government to adjust by an appropriate amount the guaranteed transfer $-T(0)$ without modifying marginal tax rates or shifting its own budget set.

Elasticities $\eta_i(w)$ reflect to what extent survival probability at age i shifts when skill increases by one percent. This set of parameters is central for determining marginal tax rates on annuities. There exist several studies which explore the empirical link between longevity and earnings (e.g. McDonough et al. (1997), Deaton and Paxson (2001) for the U.S., von Gaudecker and Scholz (2007) for Germany or Jusot (2006) for France). They all conclude that longevity is positively correlated with wage or income. They do not however frame their results in terms of income elasticity of survival rates. An exception is the paper by Bommier et al. (2005). They use a large dataset of administrative records from the French Public Pension System between 1997 and 2001 and estimate how survival rates vary with the size of pension benefits. Since the latters are almost proportional to wages in the French economy, those estimates provide a good approximation of elasticities with regard to wages. Elasticities are performed for several cohorts of retirees which vary by age and sex. They find benefit elasticities that range from 0.019 for 67 year-old men to 0.043 for 91 year-old men. The elasticities are much weaker for women. Fig. 2 shows how elasticity varies with age. The curves are linearly interpolated from their estimates provided at six different ages and for each sex. It can be seen that the elasticities increase with age for men but remain closer to zero for women. Those elasticities are used in the model by taking interpolations averaged across sex.

Finally, a wage-dependent mortality table consistent with elasticities presented in Fig. 2. is constructed in two steps. First, mortality tables by socioeconomic groups are used. They are provided by Robert-Bobée and Monteil (2005) for the French population who deceased in the middle of the 90s. The mortality table of male unskilled workers whose life expectancy is the shortest in the data is assumed to describe the mortality of the less skilled in the model (whose wage is $\underline{w}$). Next, mortality tables for higher skills are filled in by assuming that instantaneous survival probabilities at age i: $p_i(w) = \pi_i(w)/\pi_{i-1}(w)$ evolve along the wage distribution according to $p_i = kw^{\eta_i}$ where k is a parameter chosen such that $p_i(\underline{w}) = k\underline{w}^{\eta_i}$. The implied difference of life expectancy at age 65 between the less skilled $\underline{w}$ and the upper skill $\overline{w}$ is equal to six years (resp. 80.1 and 86.4 year old). As a comparison, Robert-Bobée and Monteil (2005) find a difference of four years between male unskilled workers and male executives (resp. 80.1 and 84.5 year old). This gap seems reasonable since the life expectancy of the upper skill is necessarily higher than the average life expectancy of the upper statistical group.

4.2 Numerical results

The numerical method is presented in appendix 4. Optimal marginal rates on income are plotted in Figure 3 where average income is normalized to unity. Due to a bounded distribution of skills, marginal rate is zero for the most productive agents. It is strictly positive for the least productive workers because of the existence of non-workers (Seade, 1977). Consistent with what is generally found in the literature (Mirrlees, 1971, or Tuomala, 1990), marginal rate decreases relatively fast with income.

Figure 4 shows optimal marginal tax rates on annuities at ages 70, 80 and 90. They all start from negative values, reflecting a tax-enhanced rate of return for small annuities. The tax schedule is increasing with the size of annuities, indicating a progressive pattern at all ages. To better assess how these two opposite effects work, Figure 5 compares the marginal tax rate on annuities at age 80 with the "actuarially fair" tax rate which only compensates for mortality differentials without taking into account the luxury good effect (that is $1 + t_i'(c_i(w)) = \pi_i(w)Q_i/R^{i-1}$, see (5)). The latter effect, which is represented by the gap between two same-age curves, is quantitatively small compared to the actuarial adjustment effect. Hence, the tax system is essentially designed to offset rates of return inequalities coming from life expectation differences. Note that since only a non-linear annuity tax can perform this task, a linear annuity tax would fail to capture much of the welfare gain from taxing annuities.

Figure 6 presents marginal tax rates on annuities by plotting age on the horizontal axis. Negative marginal tax rates for low-wage individuals mean that rates of return are subsidized to compensate for mortality disadvantage. Conversely, rates of return of high-wage individuals are marginally taxed up to 40% at age 90.

Tax rates become more progressive at later ages. This effect directly comes from the pattern found in the data on wage elasticities of survival probabilities. Figures 2 shows that the elasticity η_i increases with age, meaning that there is more survival rates inequality at older age. The tax system must therefore correct greater rate of return inequalities when retirees get older by way of a broader spectrum of marginal tax rates.

It is also interesting to see how the marginal tax rate pattern translates into tax transfers across wage groups. Fig. 7 keeps the convention that annuity taxes are zero for the lowest skill ($t_i(c_i(\underline{w})) = 0$, $i > 1$) and shows the ratio of tax rate to annuity $t(c_i)/(c_i+t(c_i))$ for each level of wage and at three different ages. Low annuities are subsidized with a maximum subsidy rate around the median wage. Next, annuities purchased by wages larger than twice the median wage become taxed at an increasing rate.

It has been pointed out that the main goal of an optimum annuity tax is to offset rate of return inequalities which stem from mortality heterogeneity. Yet previous results do not inform much about the magnitude of the gains for the poorest individuals or the extent of the loss for the richest individuals from implementing an annuity tax. Such an analysis is conducted by asking how much consumption a low (high) wage would accept to lose (to gain) each period in exchange of implementing an annuity tax system. Accordingly, let us denote $(c_1^*(w), c_2^*(w), ..., c_n^*(w))$, $L^*(w)$ and $U(C^*, L^*, w)$ optimum consumptions, labor supply and intertemporal utility reached by an individual with productivity w in an economy with optimum annuity taxes. Let us alternatively define $(\widehat{c}_1(w), \widehat{c}_2(w), ..., \widehat{c}_n(w))$, $\widehat{L}(w)$ and $U(\widehat{C}, \widehat{L}, w)$ the same variables in an economy *without* annuity tax but in which income tax is still set optimally under an unchanged ratio of government spending PS to aggregate production. The factor $\lambda(w)$ is a uniform consumption rate of variation in the economy with an annuity tax such that the utility levels in the two economies are equalized :

$$U((1+\lambda(w))C^*, L^*, w) = U(\widehat{C}, \widehat{L}, w). \tag{7}$$

Figure 8. shows that the scope of redistribution entailed by the annuity tax is quite significant. Implementing an annuity tax is equivalent to raising the

consumption of the poorest individual by 5% at every age and to lowering the consumption of the richest agent by 6%.

5 Conclusion

Most governments in developed countries promote retirement saving by offering tax exemptions. This paper comes to the conclusion that only the smallest annuities should be subsidized, whereas the largest annuities should be taxed. The main argument relies on the well documented fact that the rich live longer in average than the poor and therefore benefit from higher actuarial rates of return. If the level of taxation is allowed to depend on age as assumed in the model, the degree of progressivity should increase as consumers get older.

The tax system woud take a different form if the adverse selection problem were overcome by companies. In theory, a separating equilibrium may emerge if several annuity contracts endowed with different time-varying payoffs (Brunner and Pech, 2005) or different price-quantity bundles (Eckstein et al., 1985) are offered. In both cases, higher risk individuals self-select into insurance contracts that offer features that are more valuable to them than to lower risk individuals. Finkelstein and Poterba (2004) find evidence of annuitant self-selection with respect to the time profile of annuity payouts. This is however unlikely to completely eliminate the asymmetric information problem. In a companion paper, their results support the quantitative importance of adverse selection (Finkelstein and Poterba, 2002). If adverse selection problems were assumed away in the model, the luxury good effect would become the sole motive for annuity taxation.

The efficiency of the tax system is based on the fact that larger annuities are purchased by richer individuals who have systematically higher life expectancy. It could be argued however that insurers may segment the market by offering higher rates of return to those who purchase smaller annuities, thereby replicating the mechanism of the tax system. This pricing policy is however ineffective since savers have the possibility to divide their wealth between several insurers. This assumption, which excludes the case studied by Eckstein et al. (1985), and has been explicitly made when a price equilibrium in the annuity market has been defined.

The model also presumes that buying bonds is dominated by the purchase of annuities. This shortcut is very likely to be valid. The annuity taxation primarily serves to eliminate distortions in rates of return caused by longevity inequalities. Figure 5 graphically shows that the second motive of taxation, the luxury good argument, is indeed quantitatively small. As a result, the progressivity of the annuity tax schedule makes a serious dent in rates of return only for individuals who benefit from a higher rate of return due to better survival prospects. After-tax expected rates of return are pretty flat along the income axis and remain greater than the rate of return to bonds due to the mortality premium.

The present paper does not aim at presenting a definitive answer to the problem of life annuity taxation. Several improvements which are left for future work might alter its conclusions. First, a deterministic link has been assumed between income and longevity. Replacing such a link by a mere positive correlation should attenuate the level of tax progressivity. Second, it is often argued that individuals do not annuitize as often as theory would predict (Davidoff et al., 2005). This can be due to many reasons like a strong bequest motive, the presence of uncertain medical expenditures or some annuitization through state social security. Non-rational explanations have also been invoked (Brown, 2007). Insofar as a lack of annuitization is a public concern, incorporating those features in a more comprehensive model is also likely to reduce the general level of taxation.

6 Appendices

6.1 Appendix 1: The income tax

A large part of the proof is standard and its main lines can be found in Salanié (2003) or Atkinson and Stiglitz (1980). The present proof differs by the existence of a pooling annuity market, nonlinear commodity taxes and a link between the consumer's preference and skills through survival probabilities. The usual caveats concerning the lack of generality of the proof apply. In particular, it is assumed that the first-order condition does indeed characterize an optimum. I exclude the possibility that the distribution of skills results in a distribution of incomes that either has bunching at some income level or a gap in the distribution of incomes.

The first order conditions (FOC) of the consumer's program are given by:

$$\begin{aligned}\frac{U_i}{U_1} &= \frac{1+t_i'(c_i)}{Q_i} \qquad i=2,...,n \\ \frac{U_L}{U_1} &= -w(1-T')\end{aligned}$$

The FOC and the Envelope theorem lead to:

$$\mathcal{U}'(w) = U_w - \frac{LU_L}{w}$$

The budget constraint of the government can be purged from the tax schedules in the following way. Insert into the integral the resource constraint of the consumer:

$$\int_W \left(\begin{array}{c} wL - \sum_{i=1}^n (c_i + t_i(c_i))/Q_i - T(wL) \\ + \sum_{i=1}^n \pi_i t_i(c_i)/R^{i-1} + T(wL) \end{array} \right) dF(w) = PS$$

The constraint simplifies to:

$$\int_W \left(wL - \sum_{i=1}^n (c_i + t_i(c_i))/Q_i + \sum_{i=1}^n \pi_i t_i(c_i)/R^{i-1} \right) dF(w) = PS$$

Or, according to the definition (2) of Q_i to:

$$\int_W (wL - \sum_{i=1}^n \pi_i c_i/R^{i-1}) dF(w) = PS$$

Hence, the government's program can be stated as:

$$\left\{ \begin{array}{c} \max \int \Psi(\mathcal{U}(w)) dF(w) \\ \int_W (wL - \sum_{i=1}^n \pi_i c_i/R^{i-1}) dF(w) = PS \\ \mathcal{U}'(w) = -LU_L/w + U_w \end{array} \right.$$

c_i, $i = 2, ..., n$ and L are the control variables and $\mathcal{U}$ the state variable of the Hamiltonian:

$$\mathcal{H} = \Psi(\mathcal{U})f + \lambda(wL - \sum_{i=1}^n \pi_i c_i/R^{i-1})f + \mu(-\frac{LU_L}{w} + U_w) \tag{8}$$

The first date consumption c_1 is determined as an implicit function of $(c_2, ..., c_n, L, \mathcal{U})$ through the definition of $\mathcal{U}$. $L(w)$ maximizes the Hamiltonian (assuming $U_{wL} = 0$):

$$\frac{\partial \mathcal{H}}{\partial L} = \lambda f(w + \frac{U_L}{U_1}) + \mu U_L(-\frac{LU_{LL}}{U_L} - 1)/w = 0$$

Then U_L is replaced by $-w(1-T')U_1$ (FOC of the consumer's problem) and the elasticity of labor U_L/LU_{LL} by ε:

$$\lambda f w T' = -\mu(1-T')U_1\,(1+1/\varepsilon)) \tag{9}$$

or:

$$\mu/\lambda = -\frac{T'}{1-T'} wf \frac{1}{U_1(1+1/\varepsilon)} \tag{10}$$

From the definition of the Hamiltonian (8), μ varies with the wage according to (with $U_{w1} = U_{L1} = 0$):

$$\begin{aligned} \frac{d\mu}{dw} &= -\frac{\partial \mathcal{H}}{\partial \mathcal{U}} = -f\Psi' - \lambda f(-\frac{dc_1}{d\mathcal{U}}) \\ &= (\frac{\lambda}{U_1} - \Psi')f \end{aligned}$$

μ satisfies the two transversality conditions $\mu(\underline{w}) = \mu(\overline{w}) = 0$. Integrating between w and $\overline{w}$:

$$\mu(w)/\lambda = -\int_w^{\overline{w}} (\frac{1}{U_1} - \frac{\Psi'}{\lambda})dF(z) \tag{11}$$

The expression at $w = \underline{w}$ gives:

$$\mu(\underline{w})/\lambda = \int_{\underline{w}}^{\overline{w}} \left(\frac{\Psi'}{\lambda} - \frac{1}{U_1}\right) dF(z) = 0$$

or:

$$\frac{1}{\lambda} = \frac{\int_{\underline{w}}^{\overline{w}} \frac{1}{U_1} dF(z)}{\int_{\underline{w}}^{\overline{w}} \Psi' dF(z)} = \frac{E(1/U_1)}{E(\Psi')} \tag{12}$$

Substituting the left hand term of (11) in (10) leads to:

$$\frac{T'}{1-T'} = (1+1/\varepsilon)\,\frac{U_1}{wf} \int_w^{\overline{w}} \left(\frac{1}{U_1} - \frac{\Psi'}{\lambda}\right) dF(z)$$

Last, substituting λ by its expression in (12) yields the desired result:

$$\frac{T'}{1-T'} = (1+1/\varepsilon)\,\frac{U_1}{wf}\int_{w}^{\overline{w}}\left(1/U_1 - \frac{\Psi'}{E(\Psi')}E(1/U_1)\right)dF(z)$$

6.2 Appendix 2: The annuity tax

The date i consumption of an individual endowed with w maximizes the Hamiltonian (8) (assuming that $U_{w1}=0$):

$$\frac{\partial\mathcal{H}}{\partial c_i} = -\lambda f(-\frac{U_i}{U_1} + \pi_i/R^{i-1}) + \mu U_{wi} = 0$$

Replacing U_i/U_1 by $(1+t_i')\,/Q_i$ (FOC of the consumer's problem) and rearranging the terms:

$$(1+t_i')\,\frac{R^{i-1}}{\pi_i Q_i} = 1 - \frac{R^{i-1}}{\pi_i}U_{wi}\mu/\lambda f$$

Combining this equation with (9):

$$(1+t_i')\,\frac{R^{i-1}}{\pi_i Q_i} = 1 + \frac{R^{i-1}}{\pi_i}\frac{U_i}{U_1}\frac{wU_{wi}}{U_i}\frac{1}{1+1/\varepsilon}\frac{T'}{1-T'}$$

Substituting again U_i/U_1 by $(1+t_i')\,/Q_i$ leads to the expression of the annuity tax rate in terms of the income tax rate :

$$1+t_i' = \frac{\pi_i Q_i}{R^{i-1}}\left(1 - \frac{wU_{wi}}{U_i}\frac{1}{1+1/\varepsilon}\frac{T'}{1-T'}\right)^{-1}$$

The formula of the annuity tax rate can then easily be derived from the expression of the income marginal rate.

6.3 Appendix 3: Proof of Lemma 1.

Because $\pi_i(\widetilde{w})$ is a weighted average of survival rates over the whole population, it is obvious that $\pi_i(\widetilde{w}) \in \,]\pi_i(\underline{w}), \pi_i(\overline{w})[$ and that $\widetilde{w} \in \,]\underline{w}, \overline{w}[$. Next, we have from Definition (2) :

$$H(w)=\frac{\pi_i(w)\int_W (c_i(z)+t_i(c_i(z)))dF(z)}{\int_W (c_i(z)+t_i(c_i(z)))\pi_i(z)dF(z)}$$

By definition of $\widetilde{w}$, $H(\widetilde{w})=1$ and since $H'(w)=\pi_i'(w)Q_i/R^{i-1}\geq 0$, $w\geq\widetilde{w}$ ($w<\widetilde{w}$) implies $H(w)\geq 1$ ($H(w)<1$).

6.4 Appendix 4: Numerical method

This appendix presents a sketch of the numerical procedure employed. Numerical results are obtained by discretizing the interval of skills over a fine grid of points. A two-step estimation procedure is used, which is repeated until convergence. First, the consumer's problem is solved for every skill of the grid and for given tax schedules and market returns $\{T_w, T_w', t_{iw}, t_{iw}', Q_i\}$ which take values at every point of the grid and at each date. The consumers' first order conditions are :

$$\begin{aligned} c_i &= \left(\beta^{i-1}\pi_i Q_i\right)^{1/\sigma}\left(1+t_{iw}'\right)^{-1/\sigma} c_1 \\ L &= \gamma\left[w(1-T')\right]^{\varepsilon} c_1^{-\varepsilon\sigma} \end{aligned}$$

Replaced in the budget constraint:

$$\gamma w^{1+\varepsilon}(1-T_w')^{\varepsilon} c_1^{-\varepsilon\sigma}-\left[1+\sum_{j=2}^{n}\left(\frac{\beta^{j-1}\pi_j}{1+t_j'}\right)^{1/\sigma} Q_j^{(1-\sigma)/\sigma}\right] c_1-\left[T_w+\sum_{j=2}^{n} t_{jw}/Q_j\right]=0$$

This equation has one unknown c_1 and is numerically solved for each level of skill. Labor supply and consumption at later ages are retrieved from first order conditions.

Next, those values are exploited to update the vectors $\{T_w', t_{iw}', Q_i, T_w, t_{iw}\}$ by using Eq. (4), (6), (2) and the government's budget constraint (3) respectively. Standard techniques of integration are utilized to estimate the integrals.

An initial guess for the consumption and labor rules is obtained by assuming a zero value for the elasticity η. If $\eta_i = 0$, $i = 2, ...n$, the consumer's FOC becomes (see Eq. (5) for the expression of $1 + t_i'$):

$$\frac{U_i}{U_1} = \frac{1 + t_i'}{Q_i} = \frac{\pi_i(w)}{R^{i-1}}$$

The optimal consumption path of each agent is then much simpler to solve. The solution is next updated by taking positive values for η.

6.5 Appendix 5: Sensitivity analysis

The purpose of this appendix is to determine how sensitive the calibrated model's results are to changes in preference parameters' values. Three parameters are subject to variations around their baseline values: the uncompensated elasticity of labor supply ε, the relative risk aversion parameter σ (or equivalently the intertemporal rate of substitution $1/\sigma$) and the subjective discount rate β. The main quantitative features of the calibrated model are captured by four indicators which are computed for two opposite wage groups: the lowest wage quintile $Q1 = \{w; F(w) \leq 0.2\}$ and the upper wage decile $D10 = \{w; 1 - F(w) \leq 0.1\}$. Those indicators are:

1. AQ1, the average ratio of total tax burden to gross income for the lowest wage quintile:

$$\frac{1}{0.2} \int_{w \in Q1} \frac{\sum_{i=1}^{n} t_i(c_i)/Q_i + T(wL)}{wL} dF(w)$$

2. AD10, the same indicator for the upper wage decile:

$$\frac{1}{0.1} \int_{w \in D10} \frac{\sum_{i=1}^{n} t_i(c_i)/Q_i + T(wL)}{wL} dF(w)$$

3. BQ1, the average tax (or subsidy) rate on annuity at age 80 for the lowest wage quintile:

$$\frac{1}{0.2}\int_{w\in Q1}\frac{t(c_i(w))}{c_i(w)+t(c_i(w))}dF(w)$$

4. BD10, the same indicator for the upper wage decile:

$$\frac{1}{0.1}\int_{w\in D10}\frac{t(c_i(w))}{c_i(w)+t(c_i(w))}dF(w)$$

5. CQ1, the average marginal tax rate on annuities at age 80 for the lowest wage quintile:

$$\frac{1}{0.2}\int_{w\in Q1}t'(c_i(w)dF(w)$$

6. CD10, the same indicator for the upper wage decile:

$$\frac{1}{0.1}\int_{w\in D10}t'(c_i(w)dF(w)$$

7. DQ1, the average consumption rate of variation that eliminates the utility gain from implementing an annuity tax for the lowest wage quintile (see Eq. (7) for a formal definition of $\lambda(w)$):

$$\frac{1}{0.2}\int_{w\in Q1}\lambda(w)dF(w)$$

8. DD10, the average consumption rate of variation that eliminates the utility loss from implementing an annuity tax for the upper wage decile (see Eq. (7) for a formal definition of $\lambda(w)$):

$$\frac{1}{0.1}\int_{w\in D10}\lambda(w)dF(w)$$

The results are presented in the following Table (in per cent):

	Baseline (1)	$\varepsilon = 0.25$	$\varepsilon = 0.35$	$\beta = 0.95$	$\beta = 0.97$	$\sigma = 1$	$\sigma = 3$
AQ1	-70.35	-70.56	-70.38	-70.31	-70.38	-38.19	-86.74
AD10	42.56	44.31	41.07	42.55	42.57	35.20	47.14
BQ1	-0.64	-0.64	-0.64	-0.64	-0.64	-0.75	-0.59
BD10	1.55	1.55	1.55	1.55	1.55	1.34	1.75
CQ1	-18.25	-18.23	-18.28	-18.25	-18.25	-19.08	-16.86
CD10	15.33	15.33	15.33	15.33	15.33	15.20	15.44
DQ1	-4.40	-4.39	-4.41	-4.24	-4.55	-9.46	-2.61
DD10	4.14	4.00	4.00	3.88	4.14	7.09	2.66

(1) Baseline parameter values: $\varepsilon = 0.3$, $\sigma = 2$, $\beta = 0.96$.

The statistics do not vary much when the elasticity of labor supply ε or the discount rate β are changed around their baseline value. The results are somewhat more sensitive to variations of the intertemporal rate of substitution $1/\sigma$. In particular, the consumption rate of variation that eliminates the utility variation from implementing an annuity tax (DQ1 and DD10) is increasing with $1/\sigma$. This comes from the fact that, absent an annuity tax, the adverse selection effect would introduce rate of return differences across wage groups. A low intertemporal rate of substitution makes the low-wages (the high wages) more able to distort the consumption profile to offset (take advantage of) a lower (a higher) actuarial rate of return.

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Fig. 1. The empirical distribution of wage for French singles in 1995

(source : Bourguignon-Spadaro, 2007)

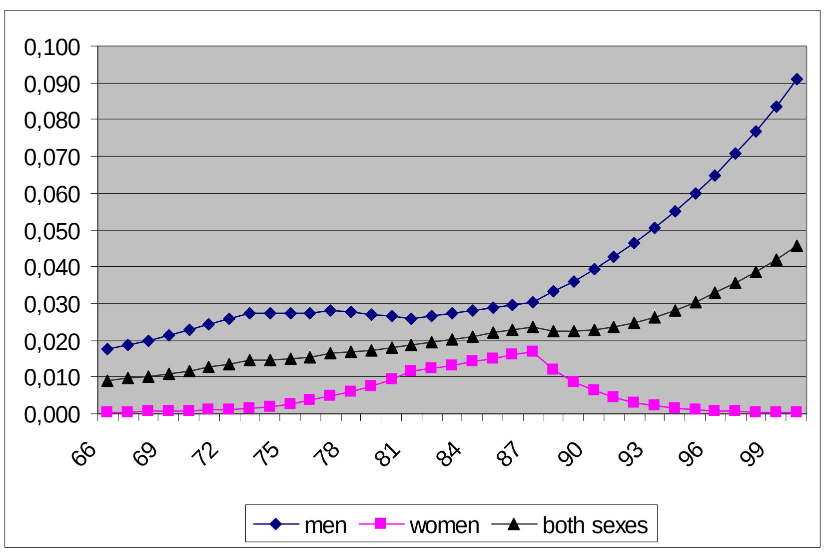

Fig. 2. Pension benefit elasticities of survival risk by age and sex

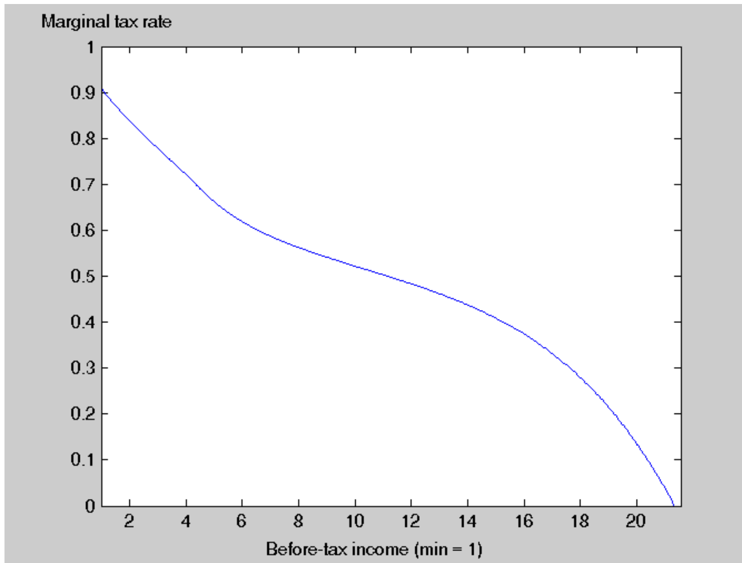

Fig. 3. The optimum marginal tax rate on income

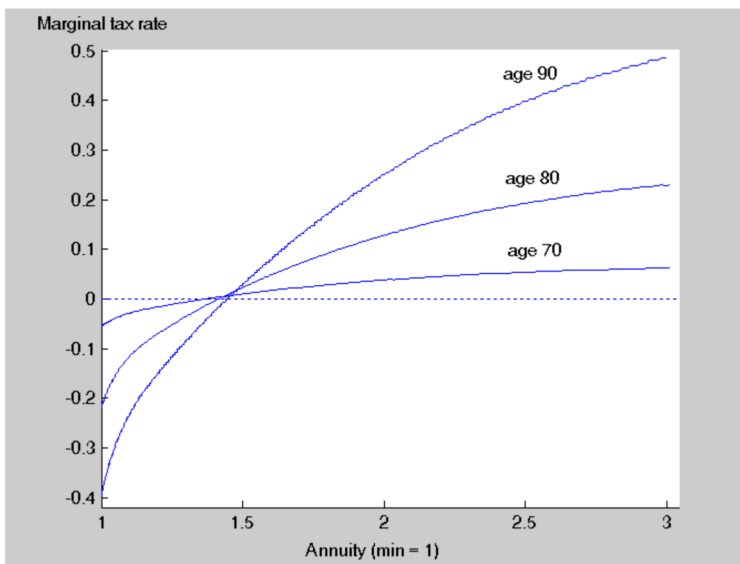

Fig. 4. Marginal tax rates on annuities at three different ages

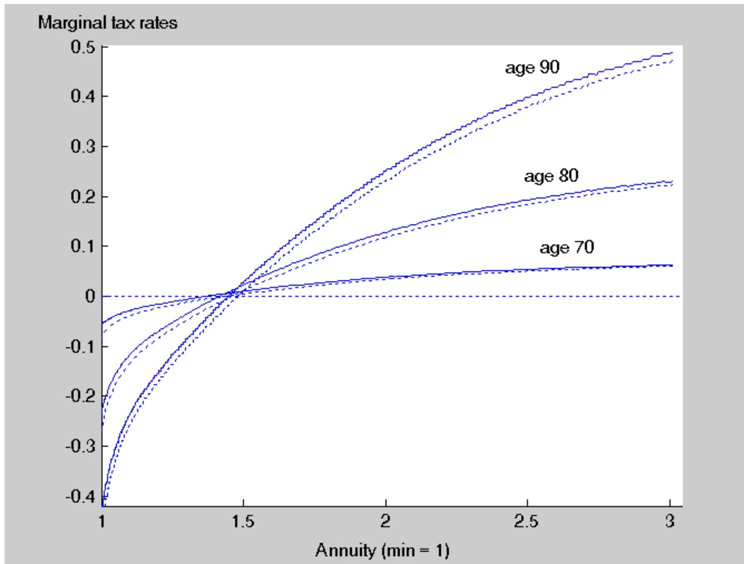

Fig. 5. Marginal tax rate on annuities (solid lines) and the "actuarially fair" tax rate (dotted lines) at three different ages.

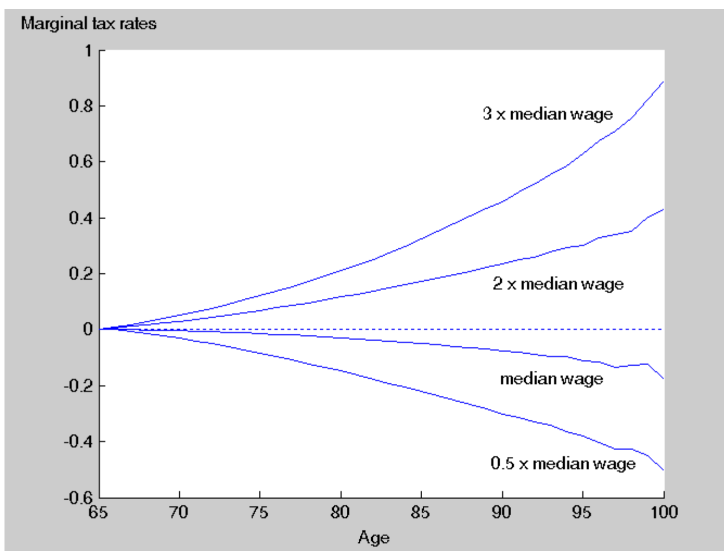

Fig. 6. Marginal tax rates on annuities and age

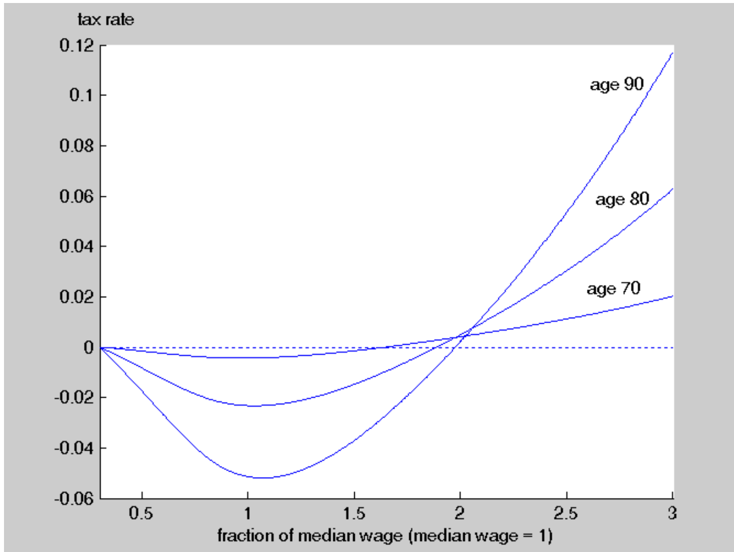

Fig. 7. Tax rate on annuities at different ages

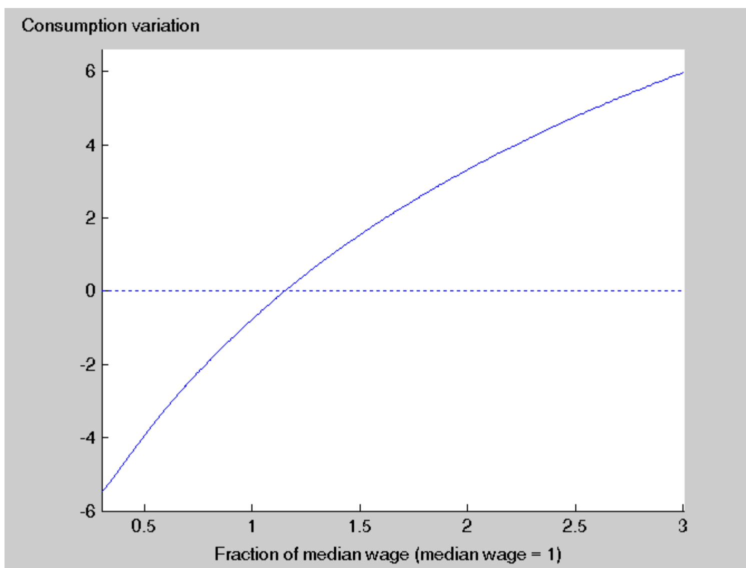

Fig. 8. Consumption variation for the economies with and without annuity tax to be equivalent