Intermittent Discounting

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Impatience is a versatile property:

People do not like waiting two minutes at a stoplight, but are willing to save for their retirement occurring in several decades.

In the first case, our attention is directed to the spotlight.

In second case, we forget most of the time that we could consume more now by saving less.

Impatience is all about withstanding the passage of time, but time is subjective and does not elapse continuously.

Aim of the paper: explore implications of intermittent waiting for time discounting.

Long tradition in psychology about the malleability of time.

William James already noted in 1890:

"The tracts of time (...) shorten in passing whenever we are so fully occupied with their content as not to note the actual time itself. (...) On the contrary, a day full of waiting, of unsatisfied desire for change, will seem a small eternity".

Stout (1932):

"In general, temporal perception is bound up with the process of attention... What measures the lapse of time is the cumulative effect of the process of attending".

New theory of time discounting

Based on the following premises:

- waiting for a reward is aversive: requires a mental effort to resist temptation and cope with some amount of frustration.
- people spend most of their time absorbed in daily activities during which future gratifications are not reminded

Example of waiting episodes:

- discussing a new model of cell-phone with a colleague
- watching a tv advertisement
- hunger, boredom, ...

Implications of intermittent waiting

The more delayed the reward, the longer the waiting period and the less valuable future utility net of waiting costs.

future utilities are discounted for two conceptually distinct reasons: the size of waiting costs and the frequency of waiting periods.

The frequency of reminding/waiting is as important as the size of discounting.

For example, repeated exposure to temptation goods may lead consumers to indulge, which is routinely exploited by the advertising industry.

I then apply the model to investigate the relation between short and long-term discounting based on two types of evidence.

First, people tend to express strong impatience over short periods (Frederick, Loewenstein and O'Donoghue, 2002).

This is problematic once the discount rates are annualized.

For example Andreoni and Sprenger (2012) estimate annualized discount rates between 25 and 38 percent.

Second, discounting is sub-additive (Read, 2001, Read and Roelofsma, 2003, Scholten and Read, 2006, Kinari et al., 2009, Dohmen et al., 2012 and Dohmen et al., 2017):

There is more overall discounting in a sequence of short-duration trade-offs than in a single trade-off over the whole interval .

I will argue that wait-based discounting is a relevant framework to understand those apparent puzzles.

Next steps of the presentation

- lay out an axiomatically founded model of consumption with intermittent waiting (Section 2)
- short-term impatience and intermittent waiting (Section 3)
- subadditive discounting and intermittent waiting (Section 4)
- conclusion (Section 5)

Time preference with waiting

I proceed in two steps. I first pose a general expected utility model of consumption and random waiting before presenting a full-fledged model of intertemporal choice.

A consumer decides at which date $t \in T = \{0, ..., \bar{t}\}$ a good, which quantity is $x \in X = (0, \bar{x}]$, is consumed.

Two types of periods

$\theta_s = 1$ if the DM pays attention to the consumption good in period s.

$\theta_s = 0$ if s is not a reminding period and consequently not a waiting period.

A reminding period is a waiting period if the good has not already been consumed in the past and is not consumed during the current period.

$\theta = (\theta_s, s = 0, ..., \bar{t}) \in \Theta = \{0, 1\}^{\bar{t}+1}$ is an exogenous temporal sequence of reminding

Expected utility with waiting

The objects of choice are lotteries with finite support. Define the set of lotteries

$$L = \left\{ P : (X, T, \Theta) \to [0, 1] \, ; \sum_{(X,T,\Theta)} P(x, t, \theta) = 1 \right\}$$

Choices expressed at time 0 are modeled by a binary relation $\succcurlyeq$ on L.

vNM axioms

A1. Weak order $\succcurlyeq$ is complete and transitive.

A2. Continuity For all P, Q, $R \in L$, if $P \succ Q \succ R$, there exist a, $b \in (0, 1)$ such that

$$aP + (1 - a)R \succ Q \succ bP + (1 - b)R$$

A3 Independence For all P, Q, $R \in L$, and $a \in (0, 1)$, $P \succcurlyeq Q$ iff

$$aP + (1 - a)R \succcurlyeq aQ + (1 - a)R$$

Theorem 1 (vNM Preference) $\succcurlyeq$ satisfies A1-A3 if and only if there exists $u_{vNM} : (X, T, \Theta) \to \mathbb{R}$ such that, for every P, $Q \in L$, $P \succcurlyeq Q$ iff

$$\sum_{(X,T,\Theta)} P(x,t,\theta)u_{vNM}(x,t,\theta) \geq \sum_{(X,T,\Theta)} Q(x,t,\theta)u_{vNM}(x,t,\theta)$$

Time preferences

I now put more structure on admissible lotteries and time preferences:

By assumption, x and t are known by the DM at date 0 hence preferences are now defined over degenerate lotteries P where x and t are certain.

The relation $\succcurlyeq'$ over dated consumptions and reminding probabilities $(x, t, p) \in X \times T \times [0, 1]^{\bar{t}+1}$ entails the same ordering as $\succcurlyeq$ with x and t certain.

Axioms on time preferences

Axiom A4 ensures that the good is valuable to the DM for any sequence of reminding probabilities:

A4. Monotonicity For all $x, y \in X$, $y > x$, $t \in T$ and $p \in [0,1]^{\bar{t}+1}$, $(y, t; p) \succ' (x, t; p)$.

The Thomsen condition ensures the multiplicative separability of the discount factor and the utility function (Fishburn and Rubinstein, 1982):

A5. Thomsen separability For all $x, y, z \in X$, $t, s, r \in T$ and $p \in [0,1]^{\bar{t}+1}$, $(x, t; p) \sim' (y, s; p)$ and $(z, t; p) \sim' (y, r; p) \Longrightarrow (x, r; p) \sim' (z, s; p)$.

Intuitively, if $y - x$ is needed to compensate for the additional delay of $s - t$, and $z - y$ for the additional delay of $t - r$, then $(y - x) + (z - y) = z - x$ is needed to compensate for the additional delay of $(s - t) + (t - r) = s - r$.

A preference between two dated consumptions with a common reminding probability is not affected by variations of this probability:

A6. Time additivity For all $x \in X$, $t, r \in T$ and $p, p', q, q' \in [0,1]^{T+1}$ such that $p' = p$ and $q' = q$ except $p'_r \neq p_r$ and $q'_r \neq q_r$, if $p_r = q_r$ then $(x, t; p) \succcurlyeq' (x, t; q) \Longrightarrow (x, t; p') \succcurlyeq' (x, t; q')$ for all $p'_r = q'_r \in [0,1]$.

The axiom ensures that waiting costs are time additively separable.

The DM is 'wait-averse'. She prefers waiting to be less likely all else equal:

A7. Waiting aversion For all $x \in X$, $t \in T$ and $p \in [0,1]^{\bar{t}+1}$, $p' = p$ except $p'_s > p_s$, $(x, t; p) \succ' (x, t; p')$ if $\forall s < t$ and $(x, t; p) \sim' (x, t; p')$ if $\forall s \geq t$.

A8. Temporal indifference For all $x \in X$, $t \in T - \{\bar{t}\}$ and $p \in [0,1]^{\bar{t}+1}$, $(x, t; p) \sim' (x,\ t+1; p)$ if $p_t = 0$.

If date t cannot be a reminding period, the DM is indifferent between consuming at this period or next one.

Intuitively, people may delay consumption effortlessly if they are distracted by unrelated activities.

For instance the DM may be willing to postpone watching the last James Bond until evening if she expects no to remind the movie during the whole day.

While waiting is only felt during reminding periods, consumption is identically valued regardless whether the period is a reminding or non-reminding state:

A9. Static indifference For all $x \in X$, $t \in T$, $p \in [0,1]^{\bar{t}+1}$, $p' = p$ except $p'_t \neq p_t$, $(x, t; p) \sim' (x, t; p')$ for all $p'_t \in [0, 1]$.

Utility representation

Axioms A1-A9 are consistent with a simple theory of discounted expected utility with waiting costs:

Proposition 1. $\succcurlyeq'$ satisfies A1-A9 if there exists $u : X \to \mathbb{R}_+$ such that, for all (x, t, p), $(y, s, q) \in X \times T \times [0, 1]^{\bar{t}+1}$, (x, t, p) $\succcurlyeq (y, s, p)$ if

$$-\sum_{j=0}^{t-1} p_j \delta_j u(x) + u(x) \geq -\sum_{j=0}^{s-1} q_j \delta_j u(y) + u(y)$$

with δ_j, $j = 0, 1, ..., T$, satisfying

$$1 > 1 - \delta_0 > 1 - \delta_0 - \delta_1 > ... > 1 - \delta_0 - ... - \delta_{T-1} > 0 \qquad (1)$$

Intertemporal utility function

$$U(x,t) = -\sum_{j=0}^{t-1} p_j \delta_j u(x) + u(x)$$

Date 0 intertemporal utility is the sum of expected waiting costs $\delta_j u(x)$ accumulated before the good is consumed and utility $u(x)$ from consuming x at date t.

Waiting costs are proportional to deferred utility $u(x)$, with the intuition that the more pleasurable the outcome, the more unpleasant the waiting.

The conditions on waiting costs:

$$1 > 1 - \delta_0 > 1 - \delta_0 - \delta_1 > ... > 1 - \delta_0 - ... - \delta_{T-1} > 0 \qquad (2)$$

Reflects both waiting aversion A7 (all $-\delta_j < 0$) and monotonicity (A4), which requires that consumption is valuable at every horizon, including in the most unfavorable environment in which the DM waits every period before consuming (all $p_j = 1$).

A two parameter version

The paper also presents a simplified and more tractable model of discounting by adding two axioms and two assumptions (see paper): $D(0) = 1$ and

$$D(t) = \pi\beta^t + (1 - \pi)\beta \tag{3}$$

with $\beta \in (0, 1)$ a discount factor and π a constant reminding probability for $t > 0$.

The smaller π (less reminding), the more patient the DM.

The model nests exponential discounting ($D(t) = \beta^t$) when the DM expects to remind the reward every period ($\pi = 1$).

Here exponential discounting signals an extreme form of impatience.

Short-term impatience

The model is applied to the relation between short and long-run rates through two related properties: impatience over short delays and subadditive discounting.

In experiments, people tend to express strong impatience over short durations (Frederick, Loewenstein and O'Donoghue, 2002).

Short-run impatience is hard to reconcile with reasonable long-run impatience due to the power of compounding.

To illustrate, consider an exponential discounter whose discount rate over a day is $\rho = 0.001$. Compounded over a full year, the long-run rate is $R = (1 + \rho)^{-365} - 1 = 44$ percent.

More reasonable levels of long-term impatience are obtained by including a bias for the present, notably the quasi-hyperbolic model of Phelps and Pollak (1968) and Laibson (1997).

However present bias models do not entirely close the gap between micro and macro estimates of time discounting.

Even after controlling for utility curvature and present bias, Andreoni and Sprenger (2012) estimate average annualized discount rates between 25 and 38 percent and Balakrishnan, Haushofer and Jakiela (2020) between 77 and 96 percent.

These values are at odds with introspection and discount rates used in macroeconomic models.

The wait-based model of discounting is consistent with short-run impatience.

Short and long-term discounting functions with waiting are:

$$D(1)/(D(0) = 1 - p_0\delta_0$$

$$D(t)/(D(0) = 1 - p_0\delta_0 - p_1\delta_1 ... - p_{t-1}\delta_{t-1}$$

Departure from perfect patience over short delays and moderate impatience over long delays can be jointly obtained if

1. the present is a reminding period (generates non-trivial impatience over short-delays)
2. subsequent episodes of reminding are infrequent ($p_1, p_2, ..., p_{t-1}$ are small)

1. is motivated by the immediate availability of the reward. Similar to a present bias: waiting is felt in the present as opposed to anticipated with some probability in the future.
2. is realistic given that most individuals spend a small fraction of their time thinking about future consumption.

Why long-run rates do not reduce to a compound of short-run rates?

Short-term impatience is generally elicited with subject's attention caught and oriented toward a concrete choice in which immediate consumption is feasible.

Yet, extrapolating long-run rates by repeatedly compounding the obtained short-run rate is like presuming that the DM is placed in the same short-term decision situation over and over.

Instead, it is more realistic to expect that she forgets the reward most of the time. This provides a strong intuition of why impatience over long-delay trade-offs is likely to remain in a reasonable range.

Subadditivity

Assume the DM is indifferent between consuming x immediately or y in t periods: $(x, 0) \sim (y, t)$.

By transitivity of indifference, there exists z such that

$$(x, 0) \sim (z, s) \sim (y, t)$$

.

With a discounted utility formulation:

$$D(0)u(x) = D(s)u(z) = D(t)u(y)$$

With the notation $\phi(s,t) = D(s)/D(t)$:

$$\phi(0,t) = \phi(0,s)\phi(s,t) \tag{4}$$

However, people tend to be more impatient when confronted with multiple short-delay trade-offs in a sub-divided interval than with a single trade-off over the whole interval (e.g. Read, 2001 or Dohmen et al., 2017):

$$\phi(0,t) < \phi(0,s)\phi(s,t) \tag{5}$$

A link exists with short-term impatience if the behavioral pattern repeats over smaller intervals. The inequality can be expanded by subdividing further every interval:

$$\begin{aligned}\phi(0,t) &< \phi(0,s)\,\phi(s,t) < \phi(0,s-1)\,\phi(s-1,s)\,\phi(s,t) \\ &< \phi(0,s-2)\,\phi(s-2,s-1)\,\phi(s-1,s)\,\phi(s,t) \\ &< \ldots \\ &< \phi(0,1)\,\phi(1,2)\ldots\phi(t-2,t-1)\,\phi(t-1,t)\end{aligned}$$

Short-run impatience and subadditive discounting share a similar property: short-duration discount rates are 'too large' to be fully consistent with long-duration discount rates.

To understand why wait-based discounting can be subadditive, consider the choice between x now or z in s periods. If preferences are additive (and for convenience multiplicatively separable), the DM chooses the late option if

$$D(0)u(x) \leq D(s)u(z)$$

Likewise, the DM prefers y in t periods to z in s periods if

$$D(s)u(z) \leq D(t)u(y)$$

In both trade-offs, utility of y is discounted by the same factor $D(t)$, which guarantees the transitivity of choice:

$$D(0)u(x) \leq D(t)u(y)$$

.

Suppose now that discounting is the result of waiting costs. In the second short-duration trade-off, the first date of the trade-off is at date s. At date 0:

- the DM anticipate the waiting costs at date s if consumption is postponed to date t
- an availability bias may operate and make waiting seem more likely (Carroll, 1978)
- the probability p_s may be objectively high if the DM anticipates that the reward will be ostensibly displayed at s.

In the long-duration trade-off between 0 and t, nothing special happens at date s, hence $p'_s < p_s$

If $p'_s < p_s$, then $D(t) < D'(t)$

$$D(t) = 1 - p_0\delta_0 - ... - p_s\delta_s - ... - p_{t-1}\delta_{t-1}$$
$$D'(t) = 1 - p_0\delta_0 - ... - p'_s\delta_s - ... - p_{t-1}\delta_{t-1}$$

The DM is more impatient in the sequence of short-trade-off in this case:

$$D(0)/D(s) \times D(s)/D(t) > D(O)/D'(t)$$

Intuitively, the DM expects less waiting episodes in a long-term trade-off than in a sequence of short-term trade-offs of same length.

A novel theory of time discounting is proposed in which the disutility of waiting explains why future utilities are depreciated.

It adopts a nonlinear approach of time, more familiar to psychologists, in which experienced time elapses only when attention to future gratifications is paid.

The theory allows to investigate important dimensions of time preferences that additive models of discounting cannot address.

When discounting is tied to expected episodes of waiting, long duration discount rates are weakly connected to short duration discount rates.

Relations between short and long rates imported from finance, like continuous discounting, compounding or annualization of subjective discount rates, should be used with caution.

If perception of durations is elastic, discounting is exposed to manipulation, a possibility explored in experiments by Mischel, Ebbesen and Raskoff Zeiss (1972) or Ebert and Prelec (2007).

A test of the theory would consist in proposing in an experiment trade-offs between a smaller reward now and a larger one later.

Subjects in the treatment group would be informed to be recalled the reward during the waiting time, for instance by watching a video related to it, by letting the reward in plain sight, or, over longer time intervals, by periodically receiving messages about it.

The theory predicts that treated subjects should express more impatience than subjects without recalls.